

Predicates and Quantifiers

Review

☐ Proposition:

1. It is a sentence that declares a fact.
2. It is either true or false, but not both.

Examples:

☐ $2 + 1 = 3.$

True Proposition

☐ Toronto is the capital of Canada.

False Proposition

☐ $x + 1 = 2.$

Neither true nor false

Review

□ Logical Operators

■ Negation

$$\neg p$$

“not p .”

■ Conjunction

$$p \wedge q$$

“ p and q .”

■ Disjunction

$$p \vee q$$

“ p or q .”

■ Exclusive or

$$p \oplus q$$

“ p or q , but not both.”

■ Conditional statement

$$p \rightarrow q$$

“If p , then q .”

■ Biconditional statement

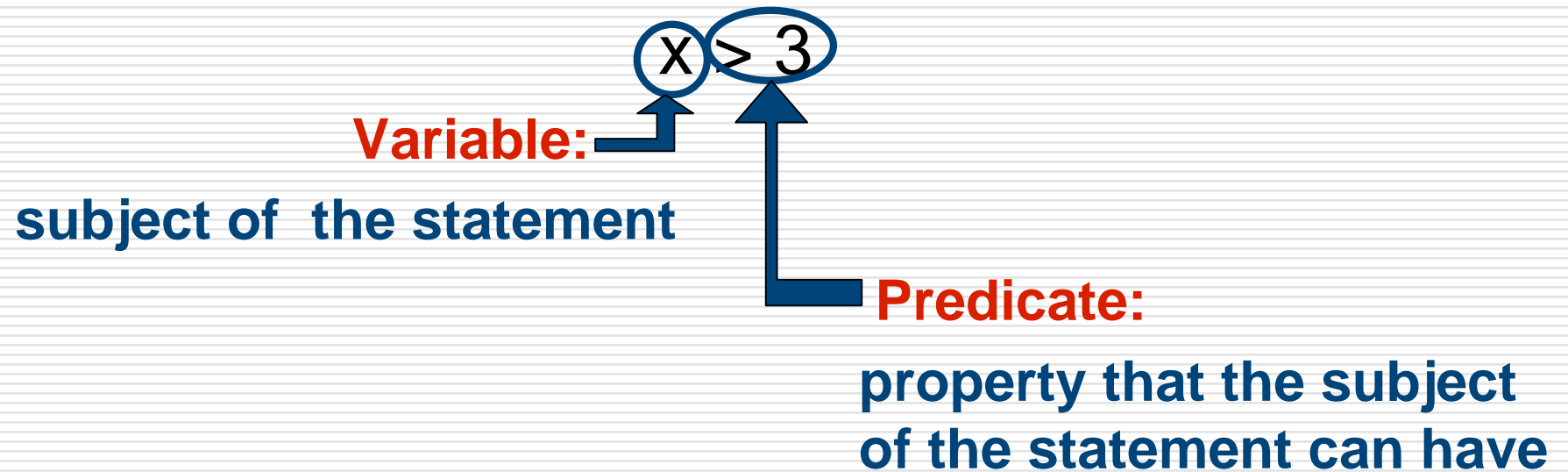
$$p \leftrightarrow q$$

“ p if and only if q .”

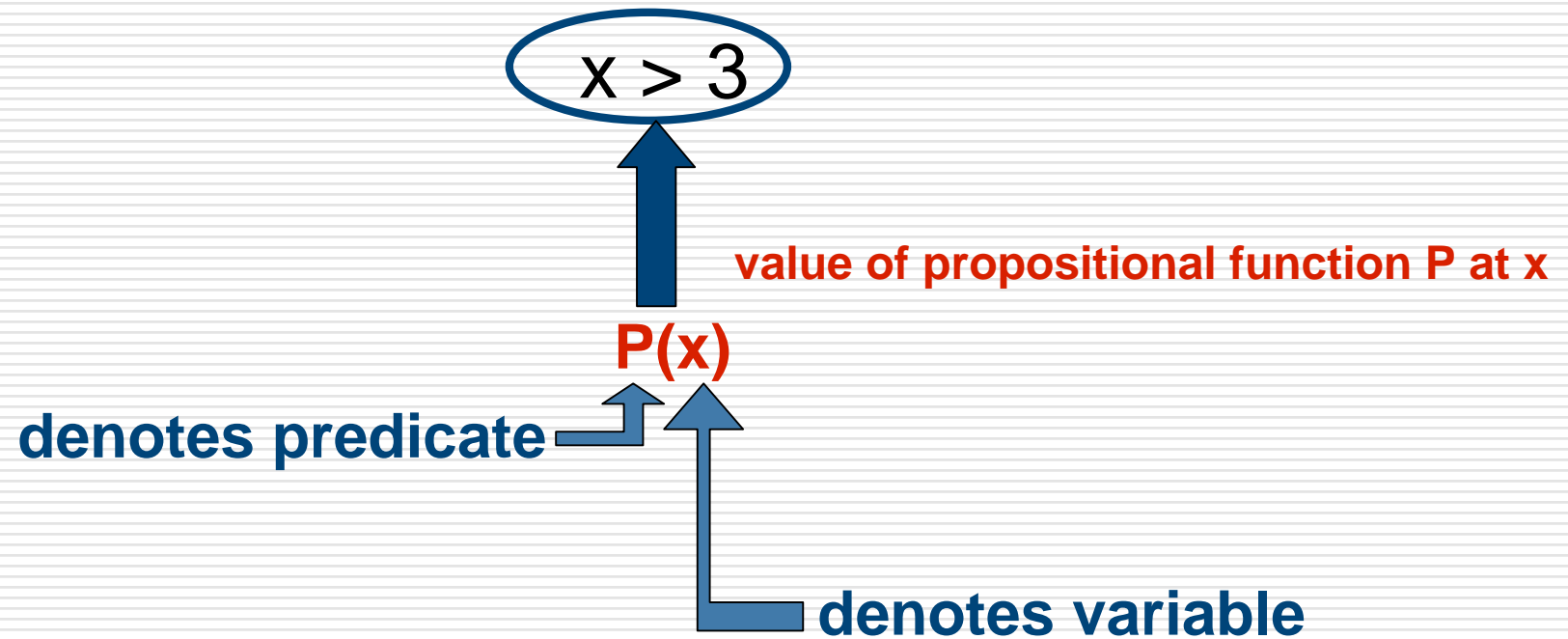
Predicate Logic

- More powerful
- Express a wide range of statements in mathematics and computer science

Predicates



Predicates



Predicates (example)

$P(x) : x > 3.$

What are the truth value of $P(4)$ and $P(2)$?

Solution:

□ Set variable x

■ $x=4$

$P(4): 4 > 3$

True

■ $x=2$

$P(2): 2 > 3$

False

Predicates (example)

$Q(x,y) : x = y+3.$

What are the truth value of $Q(1,2)$ and $Q(3,0)$?

Solution:

□ Set variables x and y

■ $x=1, y=2$

$Q(1,2): 1=2+3$

False

■ $x=3, y=0$

$Q(3,0): 3=0+3$

True

Predicates (example)

$A(c,n)$: Computer c is connected to network n .

Assume computer “CS1” is connected to network “AirYork”, but not to network “Indigo”.

What are the truth value of $A(\text{CS1}, \text{AirYork})$ and $A(\text{CS1}, \text{Indigo})$?

Solution:

□ Set variables c and n

■ $A(\text{CS1}, \text{AirYork})$:

Computer **CS1** is connected to network **AirYork**.

True

■ $A(\text{CS1}, \text{Indigo})$:

Computer **CS1** is connected to network **Indigo**.

False

Verification of computer programs

Interchange the values of two variables x and y .

1. $\text{Temp} := x$
2. $x := y$
3. $y := \text{temp}$

- ☐ Precondition: conditions that input should satisfy
 - $P(x,y): x=a, y=b$
- ☐ Postcondition: conditions that output should satisfy
 - $Q(x,y): x=b, y=a$

Verification of computer programs

Interchange the values of two variables x and y.

1. Temp := x
2. x := y
3. y := temp

□ How to verify the program?

- Assume precondition $P(x,y)$ holds

$P(x,y): x=a, y=b$ True

- Check the values of variables after each step of the program

1. Temp := x

$x=a, y=b, temp=a$

2. x := y

$x=b, y=b, temp=a$

3. y := temp

$x=b, y=a, temp=a$

- At the end, check the postcondition $Q(x,y)$

$Q(x,y): x=b, y=a$ True

Review

- Propositional function

$$P(x_1, x_2, \dots, x_n)$$

- Assign a values to variables and form proposition with certain truth value

$$Q(x,y) : x = y+3$$

$$Q(3,0) : 3 = 0+3$$

Quantifiers

- Create a proposition from a propositional function using **Quantifiers**

- Quantifiers express the **range** of elements the statement is about.
 - The universal quantifier
 - The existential quantifier

The universal quantifier

- The universal quantifier is used to assert a **property** of **all** values of a variable in a particular **domain**.

The universal quantifier

- The universal quantification of $P(x)$ is
“ $P(x)$ for all values of x in the domain.”,
denoted by $\forall x P(x)$

 **The universal quantifier**

- The universal quantifier
 - For all ...
 - For every ...
 - For each ...
 - All of ...
 - For arbitrary ...

The universal quantifier (example)

$$P(x): x+1 > x$$

The universal quantifier of $P(x)$ is in the domain of real numbers:

$$\forall x P(x) \text{ (x is a real number)}$$

$$\forall x (x+1 > x) \text{ (x is a real number)}$$

The universal quantifier

□ $\forall x P(x)$

■ When true?

□ $P(x)$ is true for every x in the domain

■ When false?

□ $P(x)$ is not always true when x is in the domain

(find a value of x that $P(x)$ is false)

The universal quantifier

- An element for which $P(x)$ is false is called a **counterexample** of $\forall x P(x)$.

$P(x): x > 3$

$P(2): 2 > 3$ is a counterexample of $\forall x P(x)$

The universal quantifier (example)

$P(x): x+1 > x.$

What is the truth value of $\forall x P(x)$ in the domain of real numbers?

Solution:

□ Check if $P(x)$ is true for all real numbers

■ “ $x+1 > x$ ” is true for all real number

So, the truth value of $\forall x P(x)$ is true.

The universal quantifier (example)

$Q(x): x < 2.$

What is the truth value of $\forall x Q(x)$ in the domain of real numbers?

Solution:

□ Find a counterexample for $\forall x Q(x)$

■ $Q(3): 3 < 2$ is false

$x=3$ is a counterexample for $\forall x Q(x)$, so $\forall x Q(x)$ is false.

The universal quantifier (example)

$P(x): x^2 > 0.$

What is the truth value of $\forall x P(x)$ in the domain of integers?

Solution:

□ Find a counterexample for $\forall x P(x)$

■ $P(0): 0 > 0$ is false

$x=0$ is a counterexample for $\forall x P(x)$, so $\forall x P(x)$ is false.

The universal quantifier (example)

$A(x)$: Computer x is connected to the network.

What is $\forall x A(x)$ in the domain of all computers on campus?

Solution:

□ $\forall x A(x)$:

“For every computer x on campus, computer x is connected to the network.”

“Every computer on campus is connected to the network.”

The universal quantifier (example)

$P(x): x^2 \geq x$.

What is the truth value of $\forall x P(x)$ in the domain of all real numbers?

Solution:

□ Find a counterexample for $\forall x P(x)$

■ $P(1/2): 1/4 \geq 1/2$ is false

$x=1/2$ is a counterexample for $\forall x P(x)$, so $\forall x P(x)$ is false.

The universal quantifier (example)

$P(x): x^2 \geq x$.

What is the truth value of $\forall x P(x)$ in the domain of all real numbers?

Solution: How to find a counterexample?

$$x^2 \geq x.$$

$$(x^2 - x) = x(x - 1) \geq 0.$$

| |
|--|
| x and $(x-1)$ must both be zero or positive. |
|--|

| |
|---------------------------------|
| $x \geq 0$ and $(x - 1) \geq 0$ |
|---------------------------------|

| |
|---------------------------|
| $x \geq 0$ and $x \geq 1$ |
|---------------------------|

| |
|------------|
| $x \geq 1$ |
|------------|

OR

| |
|--|
| x and $(x-1)$ must both be zero or negative. |
|--|

| |
|---------------------------------|
| $x \leq 0$ and $(x - 1) \leq 0$ |
|---------------------------------|

| |
|---------------------------|
| $x \leq 0$ and $x \leq 1$ |
|---------------------------|

| |
|------------|
| $x \leq 0$ |
|------------|

$0 < x < 1$ such as $x=1/2$ is a counterexample

The universal quantifier (example)

$P(x): x^2 \geq x.$

What is the truth value of $\forall x P(x)$ in the domain of all integers?

Solution:

- Check if $P(x)$ is true for all integers
 - $P(x)$ is true when $x \geq 1$ or $x \leq 0$.
 - There is no integer between $0 < x < 1$.
 - So, $\forall x P(x)$ is true for the domain of all integers.

The universal quantifier

- $\forall x P(x)$ in the domain D
- If D can be listed as x_1, x_2, \dots, x_n .

$\forall x P(x)$ in the domain D is the same as
 $P(x_1) \wedge P(x_2) \wedge \dots \wedge P(x_n)$

The universal quantifier (example)

$P(x): x^2 < 10.$

What is the truth value of $\forall x P(x)$ in the domain of positive integers not exceeding 4?

Solution:

- List the domain
 - Domain is 1, 2, 3, 4.
- Find the equivalent conjunction and its truth value
 - $P(1) \wedge P(2) \wedge P(3) \wedge P(4)$
 - $T \wedge T \wedge T \wedge F$ which is false
- So, $P(4)$ is a counterexample and $\forall x P(x)$ is false.

The existential quantifier

- The existential quantifier is used to assert a **property of at least one** value of a variable in a **domain**.

The existential quantifier

- The existential quantification of $P(x)$ is
“There exists an element x in the domain such that
 $P(x)$.”,
denoted by $\exists x P(x)$

 **The existential quantifier**

- The existential quantifier
 - There exists ...
 - There is ...
 - For some ...
 - For at least one ...

The existential quantifier (example)

$$P(x): x > 3$$

The existential quantifier of $P(x)$ is in the domain of integers:

$$\exists x P(x) \text{ (x is an integer)}$$

$$\exists x (x > 3) \text{ (x is an integer)}$$

The existential quantifier

□ $\exists x P(x)$

■ When true?

□ There is an x for which $P(x)$ is true.
(find a value of x that $P(x)$ is true.)

■ When false?

□ $P(x)$ is false for every x .

The existential quantifier (example)

$P(x): x > 3.$

What is the truth value of $\exists x P(x)$ in the domain of real numbers?

Solution:

□ Check if $P(x)$ is true for some real numbers

■ “ $x > 3$ ” is true when $x = 4$.

So, the truth value of $\exists x P(x)$ is true.

The existential quantifier (example)

$Q(x): x = x+1.$

What is the truth value of $\exists x Q(x)$ in the domain of real numbers?

Solution:

□ Check if $Q(x)$ is false for all real numbers

■ “ $x = x+1$ ” is false for all real numbers.

So, the truth value of $\exists x Q(x)$ is false.

The existential quantifier

- $\exists x P(x)$ in the domain D
- If D can be listed as x_1, x_2, \dots, x_n .

$\exists x P(x)$ in the domain D is the same as
 $P(x_1) \vee P(x_2) \vee \dots \vee P(x_n)$

The existential quantifier (example)

$P(x): x^2 > 10.$

What is the truth value of $\exists x P(x)$ in the domain of positive integers not exceeding 4?

Solution:

- ☐ List the domain
 - Domain is 1, 2, 3, 4.
- ☐ Find the equivalent disjunction and its truth value
 - $P(1) \vee P(2) \vee P(3) \vee P(4)$
 - $F \vee F \vee F \vee T$ which is true.
- ☐ So, $\exists x P(x)$ is true.

Quantifiers (review)

| Statement | When True? | When False? |
|------------------|---|--|
| $\forall x P(x)$ | $P(x)$ is true for every x . | There is an x for which $P(x)$ is false. |
| $\exists x P(x)$ | There is an x for which $P(x)$ is true. | $P(x)$ is false for every x . |

Translating from English into logical expression (example)

Express the following statement using predicates and quantifiers?

“Every student in this class has studied calculus.”

Solution:

- Determine individual propositional function
 - $P(x)$: x has studied calculus.
- Translate the sentence into logical expression
 - $\forall x P(x)$ domain: students in class

Translating from English into logical expression (example)

Express the following statement using predicates and quantifiers?

“Some student in this class has visited Mexico.”

Solution:

- Determine individual propositional function
 - $P(x)$: x has visited Mexico.
- Translate the sentence into logical expression
 - $\exists x P(x)$ domain: students in class

Translating from English into logical expression (example)

Express the following statement using predicates and quantifiers?

“Every student in this class has visited either the US or Mexico.”

Solution:

- Determine individual propositional functions
 - $P(x)$: x has visited the US.
 - $Q(x)$: x has visited Mexico.
- Translate the sentence into logical expression
 - $\forall x (P(x) \vee Q(x))$ domain: students in class

Quantifiers with restricted domain

- Sometimes a variable in the domain must satisfy a **condition**.

Quantifiers with restricted domain (example)

$$\forall x < 0 (x^2 > 0).$$

What does this statement mean in the domain of real numbers?
(express it in English and logic using conditional statement)

Solution:

- Express the statement in English
 - For every real number x with $x < 0$, $x^2 > 0$.
 - The square of a negative real number is positive.
- Find the equivalent if statement (find condition and property)
 - Condition: $x < 0$ property: $x^2 > 0$
 - $\forall x (x < 0 \rightarrow x^2 > 0)$

Quantifiers with restricted domain (example)

$$\forall y \neq 0 (y^3 \neq 0).$$

What does this statement mean in the domain of real numbers?
(express it in English and logic using conditional statement)

Solution:

- Express the statement in English
 - For every real number y with $y \neq 0$, $y^3 \neq 0$.
 - The cube of every nonzero real number is nonzero.
- Find the equivalent if statement (find condition and property)
 - Condition: $y \neq 0$ property: $y^3 \neq 0$
 - $\forall y (y \neq 0 \rightarrow y^3 \neq 0)$

Quantifiers with restricted domain (example)

$$\exists z > 0 (z^2 = 2).$$

What does this statement mean in the domain of real numbers?
(express it in English and logic using conjunction)

Solution:

- Express the statement in English
 - There exists a real number z with $z > 0$, $z^2 = 2$.
 - There is a positive square root of 2.
- Find the equivalent conjunction (find conditions)
 - Conditions: $z > 0$ $z^2 = 2$
 - $\exists z (z > 0 \wedge z^2 = 2)$

Example of system specifications

Express the following system specifications.

“Every mail message larger than one megabyte will be compressed.”

“If a user is active, at least one network link will be available.”

Solution:

- Determine individual predicates
 - Mail message is larger than one megabyte.
 - Mail message will be compressed.
 - User is active.
 - Network link will be available.

Example of system specifications

“Every mail message larger than one megabyte will be compressed.”

“If a user is active, at least one network link will be available.”

Solution:

- Introduce variables for each sentence
 - Mail message is larger than one megabyte.
 - **$P(m,x)$: Mail message m is larger than x megabytes.**
domain of m : **all mail messages**
domain of x : **positive real numbers**
 - Mail message will be compressed.
 - **$Q(m)$: Mail message m will be compressed.**
domain of m : **all mail messages**

Example of system specifications

“Every mail message larger than one megabyte will be compressed.”

“If a user is active, at least one network link will be available.”

Solution:

- Introduce variables for each sentence
 - User is active.
 - **R(u): User u is active.**
domain of u: **all users**
 - Network link will be available.
 - **S(n): Network link n is available.**
domain of n: **all network links**

Example of system specifications

“Every mail message larger than one megabyte will be compressed.”

“If a user is active, at least one network link will be available.”

Solution:

- Translate each specification into logical expression
 - “Every mail message larger than one megabyte will be compressed.”
 - $P(m,x)$: Mail message m is larger than x megabytes.
 - $Q(m)$: Mail message m will be compressed.

$$\forall m (P(m,1) \rightarrow Q(m))$$

Example of system specifications

“Every mail message larger than one megabyte will be compressed.”

“If a user is active, at least one network link will be available.”

Solution:

- Translate each specification into logical expression
 - “If a user is active, at least one network link will be available.”
 - $R(u)$: User u is active.
 - $S(n)$: Network link n is available.

$$\exists u R(u) \rightarrow \exists n S(n)$$

Example

Express the following sentences using predicates and quantifiers.

“All lions are fierce.”

“Some lions do not drink coffee.”

Assume

$P(x)$: x is a lion.

$Q(x)$: x is fierce.

$R(x)$: x drinks coffee.

Solution:

□ Translate each sentence into logical expression

■ “All lions are fierce.”

□ $\forall x (P(x) \rightarrow Q(x))$

Example

Express the following sentences using predicates and quantifiers.

“All lions are fierce.”

“Some lions do not drink coffee.”

Assume

$P(x)$: x is a lion.

$Q(x)$: x is fierce.

$R(x)$: x drinks coffee.

Solution:

□ Translate each sentence into logical expression

■ “Some lions do not drink coffee.”

□ $\exists x (P(x) \wedge \neg R(x))$

Precedence of quantifiers

The quantifiers \forall and \exists has higher precedence than all logical operators.

Example: $\forall x p(x) \vee Q(x)$
 $(\forall x p(x)) \vee Q(x)$

| Operators | Precedence |
|-------------------|------------|
| $\forall \exists$ | 0 |
| \neg | 1 |
| \wedge | 2 |
| \vee | 3 |
| \rightarrow | 4 |
| \leftrightarrow | 5 |

Binding variables

- ☐ Variable

 - Bound

 - ☐ Quantifiers

 - Free

 - ☐ Not bound

- ☐ Turn a propositional function into a proposition

 - All variables must be **bound**.

Binding variables (example)

$\exists x (x+y=1).$

Is it a proposition?

Solution:

- ☐ Check if any variable is free
 - Variable x
 - ☐ bound
 - Variable y
 - ☐ Free
 - Since variable y is free, it is not a proposition.

Logical equivalences

- Assume S and T are two statements involving predicates and quantifiers.
- S and T are **logically equivalent** if and only if they have the same truth value no matter which predicates are substituted and which *domain is used* for the variables, denoted by $S \equiv T$.

Logical equivalences (example)

Show that $\forall x (P(x) \wedge Q(x))$ and $\forall x P(x) \wedge \forall x Q(x)$ are logically equivalent.

Solution:

Part1: Show if $\forall x (P(x) \wedge Q(x))$ is true then $\forall x P(x) \wedge \forall x Q(x)$ is true. (using direct technique)

- Assume $\forall x (P(x) \wedge Q(x))$ is true.
- If a is in the domain, then $P(a) \wedge Q(a)$ is true.
- So, $P(a)$ is true and $Q(a)$ is true.
- Since $P(a)$ and $Q(a)$ are both true for every element in the domain, $\forall x P(x)$ and $\forall x Q(x)$ are both true.
- So, $\forall x P(x) \wedge \forall x Q(x)$ is true.

Logical equivalences (example)

Show that $\forall x (P(x) \wedge Q(x))$ and $\forall x P(x) \wedge \forall x Q(x)$ are logically equivalent.

Solution:

Part2: Show if $\forall x P(x) \wedge \forall x Q(x)$ is true then $\forall x (P(x) \wedge Q(x))$ is true. (using direct technique)

- Assume $\forall x P(x) \wedge \forall x Q(x)$ is true.
- So, $\forall x P(x)$ is true and $\forall x Q(x)$ is true.
- If a is in the domain, then $P(a)$ is true and $Q(a)$ is true.
- If $P(a)$ is true and $Q(a)$ is true, then $P(a) \wedge Q(a)$ is true.
- Since $P(a) \wedge Q(a)$ is true for every element in the domain, $\forall x (P(x) \wedge Q(x))$ is true.

□ So, $\forall x (P(x) \wedge Q(x)) \equiv \forall x P(x) \wedge \forall x Q(x)$.

Logical equivalences (example)

Show that $\forall x (P(x) \vee Q(x))$ and $\forall x P(x) \vee \forall x Q(x)$ are not logically equivalent.

Solution:

- Give an example that $\forall x (P(x) \vee Q(x))$ and $\forall x P(x) \vee \forall x Q(x)$ have different truth values.
 - $P(x)$: x is odd. $Q(x)$: x is even. (in the domain of integers.)
 - For all element $(P(x) \vee Q(x))$ is true. (all x is odd or even.)
 - So, $\forall x (P(x) \vee Q(x))$ is true.
 - For all element $P(x)$ is false. (all x is not odd.)
 - For all element $Q(x)$ is false. (all x is not even.)
 - So, $\forall x P(x) \vee \forall x Q(x)$ is false.
- Thus, $\forall x (P(x) \vee Q(x))$ and $\forall x P(x) \vee \forall x Q(x)$ are not logically equivalent.

Negation (review)

Let p be a proposition.

The **negation of p** , denoted by $\neg p$, is the proposition “It is not the case that p .”

Negating quantified expression

| Statement | When False? |
|------------------|--|
| $\forall x P(x)$ | There is an x for which $P(x)$ is false. |
| $\exists x P(x)$ | $P(x)$ is false for every x . |

| | |
|-----------------------|-----------------------|
| $\neg \forall x P(x)$ | $\exists x \neg P(x)$ |
| $\neg \exists x P(x)$ | $\forall x \neg P(x)$ |

Negating quantified expression

□ Assume $\forall x P(x)$ is:

“Every student has taken a course in calculus.”

□ $\neg (\forall x P(x))$ is:

“It is not the case that every student has taken a course in calculus.”

“There is a student who has not taken a course in calculus.”

$\exists x \neg P(x)$

$$\neg (\forall x P(x)) \equiv \exists x \neg P(x)$$

Negating quantified expression

Show that $\neg (\forall x P(x))$ and $\exists x \neg P(x)$ are logically equivalent.

Solution:

Show $\neg \forall x P(x)$ is true if and only if $\exists x \neg P(x)$ is true. (using direct technique)

- $\neg \forall x P(x)$ is true if and only if $\forall x P(x)$ is false.
- $\forall x P(x)$ is false if and only if there is an element in the domain for which $P(x)$ is false.
- There is an element for which $P(x)$ is false if and only if there is an element for which $\neg P(x)$ is true.
- There is an element for which $\neg P(x)$ is true if and only if $\exists x \neg P(x)$ is true.

□ Thus, $\neg (\forall x P(x))$ and $\exists x \neg P(x)$ are logically equivalent.

Negating quantified expression

□ Assume $\exists x P(x)$ is:

“There is a student who has taken a course in calculus.”

□ $\neg (\exists x P(x))$ is:

“It is not the case that there is a student who has taken a course in calculus.”

“Every student has not taken a course in calculus.”

$\forall x \neg P(x)$

$$\neg (\exists x P(x)) \equiv \forall x \neg P(x)$$

Negating quantified expression

Show that $\neg (\exists x P(x))$ and $\forall x \neg P(x)$ are logically equivalent.

Solution:

Show $\neg \exists x P(x)$ is true if and only if $\forall x \neg P(x)$ is true. (using direct technique)

- $\neg \exists x P(x)$ is true if and only if $\exists x P(x)$ is false.
- $\exists x P(x)$ is false if and only if there is no element in the domain for which $P(x)$ is true.
- There is no element for which $P(x)$ is true if and only if for all elements $\neg P(x)$ is true.
- For all elements $\neg P(x)$ is true if and only if $\forall x \neg P(x)$ is true.

□ Thus, $\neg (\exists x P(x))$ and $\forall x \neg P(x)$ are logically equivalent.

De Morgan's laws for quantifiers

| Negation | Equivalent st. | When true? | When false? |
|-----------------------|-----------------------|---------------------------------------|--------------------------------------|
| $\neg \forall x P(x)$ | $\exists x \neg P(x)$ | There is an x that $P(x)$ is false. | For all x $P(x)$ is true. |
| $\neg \exists x P(x)$ | $\forall x \neg P(x)$ | For all x $P(x)$ is false. | There is an x that $P(x)$ is true. |

De Morgan's laws for quantifiers

- $\forall x P(x)$ in the domain D
- If D can be listed as x_1, x_2, \dots, x_n .
- $\forall x P(x) \equiv$
 $P(x_1) \wedge P(x_2) \wedge \dots \wedge P(x_n)$
- $\neg \forall x P(x) \equiv$
 $\neg(P(x_1) \wedge P(x_2) \wedge \dots \wedge P(x_n)) \equiv$
 $\neg P(x_1) \vee \neg P(x_2) \vee \dots \vee \neg P(x_n)$

De Morgan's laws for quantifiers

- $\exists x P(x)$ in the domain D
- If D can be listed as x_1, x_2, \dots, x_n .
- $\exists x P(x) \equiv$
 $P(x_1) \vee P(x_2) \vee \dots \vee P(x_n)$
- $\neg \exists x P(x) \equiv$
 $\neg(P(x_1) \vee P(x_2) \vee \dots \vee P(x_n)) \equiv$
 $\neg P(x_1) \wedge \neg P(x_2) \wedge \dots \wedge \neg P(x_n)$

Negating quantified expression (example)

What is the negation of the statement “All Canadians eat sushi”?

Solution:

- Determine individual propositional function
 - $P(x)$: x eats sushi.
- Then translate the sentence into logical expression
 - $\forall x P(x)$ domain: Canadians
- Find the negation of $\forall x P(x)$
 - $\exists x \neg P(x)$ domain: Canadians
- Translate $\exists x \neg P(x)$ into English sentence
 - Some Canadian does not eat sushi.

Negating quantified expression (example)

What is the negation of the statement “There is an honest politician”?

Solution:

- Determine individual propositional function
 - $P(x)$: x is an honest politician.
- Then translate the sentence into logical expression
 - $\exists x P(x)$ domain: politicians
- Find the negation of $\exists x P(x)$
 - $\forall x \neg P(x)$ domain: politicians
- Translate $\forall x \neg P(x)$ into English sentence
 - Every politician is dishonest.

Negating quantified expression (example)

What is the negation of $\forall x (x^2 > x)$?

Solution:

$$\neg(\forall x (x^2 > x)) \equiv$$

$$\exists x \neg(x^2 > x) \equiv$$

$$\exists x (x^2 \leq x)$$

Negating quantified expression (example)

What is the negation of $\exists x (x^2 = 2)$?

Solution:

$$\neg(\exists x (x^2 = 2)) \equiv$$

$$\forall x \neg(x^2 = 2) \equiv$$

$$\forall x (x^2 \neq 2)$$

Recommended exercises

5,10,13,19,21,23,25,33,35,39,61